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Uncertainty evolution and its estimation with respect to the atmosphere-ocean system

Understanding the uncertainty of a state is not only important in assessing the forecast of that state, but also of interest in its own right from a physical point of view. Within the information theoretic framework, uncertainty is conveniently measured in terms of Shannon entropy, which we denote by H henceforth. For the atmosphere-ocean system, however, H is in general computationally intractable due to its huge dimensionality. Liang (2011) argued that this actually can be circumvented by studying *dH/dt*, rather than H itself, on the basis of a side finding in Liang and Kleeman (2005). The finding is best expressed as a formula: *dH/dt = E(gradF)*, where E stands for the operator of mathematical expectation, and **F** for the vector field of the dynamical system under consideration, which in this context can be viewed as the atmosphere/ocean model. A comprehensive and systematic derivation with both deterministic and stochastic systems has been made (Liang, 2014), bringing connections between the two physical notions, namely, uncertainty and instability. It is interesting to note that the Lorenz system and stochastic flow system are both examples of self-organization in the light of uncertainty reduction. The latter particularly shows that, sometimes stochasticity may actually enhance the self-organization process.
More often than not, we are more interested in, for a forecast, its local performance, rather than its overall performance. In this case, the above law does not help much; what we need to know is the local marginal entropy evolution. This requires a solution of the entailed probability density equation, which is in general computationally intractable. Here we show that, from a data science point of view, it actually can be estimated through maximum likelihood estimation. The resulting formulas are rather concise in form, stating that the marginal entropy evolution of a variable at a point is equal to the rate of the local uncertainty generation, plus the transference of the uncertainties from all other locations. This has also been connected to another systematic work of the author, i.e., quantitative causality analysis ab initio (e.g., Liang 2016; a recent short review is referred to Liang et al., 2023).
As a demonstration, shown in Fig. 1 are the uncertainty evolutions of the 2015 "monster"El Niño and South China Sea surface circulation based on the ECCO SST data. Since only positive entropy implies uncertainty, the negative values are not contoured here. For the El Niño case (Fig. 1a), uncertainty is maximized in spring. This is the well-known spring predictability barrier. Something overlooked before is the second maximum in the fall of 2015, though much weaker than its spring counterpart. For the South China Sea case (Fig. 1b), one easily sees the maximum in fall south of Vietnam, a fall predictability barrier which has been identified before in operational forecasts.

Keywords: uncertainty; entropy; second law of thermodynamics; information flow; ensemble prediction; quasi-geostrophic flow; Fisher information matrix; self-organization; El Niño; spring predictability barrier; South China Sea

Fig. 1. Uncertainty evolution of (a) the 2015 "Monster"El Niño and (b) South China Sea surface circulation

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